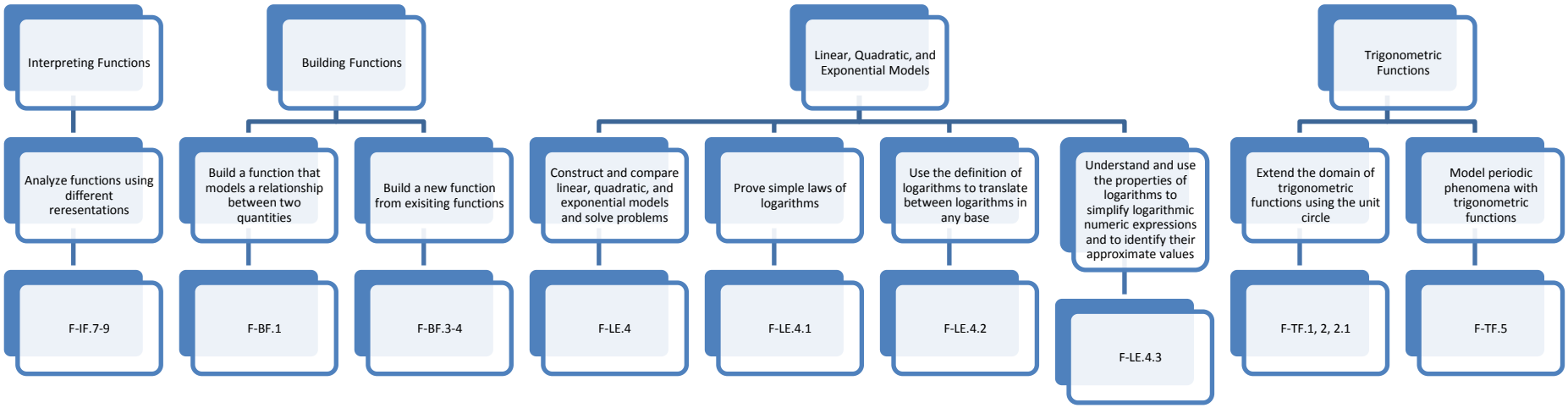
Honors Advanced Mathematics
Unit 2
Functions and Trigonometry



Honors Advanced Mathematics – UNIT 2
Functions, Logarithms and Trigonometry

Critical Area:

Students will develop the general understanding of functions in terms of their behavior and the properties including increasing and decreasing functions, concavity, even / odd functions, end behavior and asymptotes . They synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They graph shapes and relate the graphs to the behavior of the functions with the transformation on the variable (e.g. the graph of $y = f\ (x + 2)$).

Students expand their understanding of the trigonometric functions first developed in Geometry to explore the graphs of trigonometric functions with attention to the connection between the unit circle representation of the trigonometric functions and their properties, use trigonometric functions to model periodic phenomena.

CLUSTERS	COMMON CORE STATE STANDARDS
(m) Analyze functions using different representations	**F-IF: Interpreting Functions** **F-IF.7**. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise - defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.9.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)
(m) Building a function that models a relationship between two quantities	**F-BF: Building Functions** **F-BF.1** Write a function that describes a relationship between two quantities. ★ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

(s) Building functions from existing functions	**F-BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-BF.4** Find inverse functions. a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x³ or f(x) = (x + 1)/(x − 1) for x ≠ 1*
(s/a)Construct and compare linear, quadratic, and exponential models and solve problems	**F-LE: Linear, Quadratic, and Exponential Models**★ **F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. [Logarithms as solutions for exponentials.] 4.1 Prove simple laws of logarithms. CA ★ 4.2 Use the definition of logarithms to translate between logarithms in any base. CA ★ 4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. CA ★
(s/a)Extend the domain of trigonometric functions using the unit circle.	**F-TF: Trigonometric Functions** **F-TF.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.2.** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF. 2.1** Graph all 6 basic trigonometric functions. CA ★
(s/a)Model periodic phenomena with trigonometric functions.	**F-TF. 5**. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. CA ★

MATHEMATICAL PRACTICES	PROGRESSION
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the arguments of others. 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.**	http://opi.mt.gov/PDF/CCSSO/MCCS-MATH/STAGE1/Resources/2012_12-04Draft-High-School-Progression-Functions.pdf http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_modeling_2013_07_04.pdf

7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
Functions: • Different types of relationships between quantities can be modeled with different types of functions. • Functions and relations can be represented using polar coordinates. • Functions and equations can be defined parametrically. • All functions have algebraic, numerical, graphical and verbal representations. • Operations and transformations apply to all types of functions and can be used to build new functions from existing functions. • The inverse functions interchange the domain and the range. • The domain of a non-invertible function needs to be restricted in order to construct its inverse function. • Graphs of functions can explain the observed local and global behavior of a function. • Asymptotes represent the restricted domain or range. • The graph of a function demonstrates the end behavior as it approaches the vertical, horizontal or oblique asymptotes. • Real world situations can be modeled and solved by using various functions. **Logarithms:** • Logarithms are exponents. • Logarithms are used to solve exponential	1. What relationships exist between quantities that can be modeled by functions? 2. How can functions and relations be represented using polar coordinates? 3. Why is it important to define functions and equations parametrically? 4. What does it mean to solve equations graphically? 5. What do the domain and the range of a function represent? 6. What do asymptotes represent? 7. What do the maximum and minimum represent and how do they relate to the end behavior of a function? 8. How do we build new functions from existing functions using transformations? 9. What are the similarities and differences between linear, quadratic, exponential, logarithmic and polynomial functions? 10. How do we compare/contrast exponential and logarithmic functions? 11. What are inverse functions and what are they being used for? 12. How do we restrict the domain of a non-invertible function to produce an invertible function? 13. How can we use logarithms to solve an equation when the exponent is a variable? How can we change the base of a logarithm? 14. What is the angle of rotation, and how is it measured? 15. Why do we need radian measure? 16. How can sine, cosine, and tangent functions be defined using the unit circle?	• Asymptotes - horizontal, vertical and oblique • Complex roots • Composite function • Compress/ stretch • Domain/ Range • End behavior • Even/ odd functions • Exponential • Frequency • Increasing/decreasing • Intercepts • Inverse function • Invertible, non-invertible • Laws of Logarithms • Logarithmic expressions • Logarithmic functions • Maximum/ minimum • Midline • One-to-one functions • Period, amplitude, phase shift • Periodicity • Piecewise function • Quadrantal and coterminal angles

equations. • The definition of logarithms can be used to translate between logarithms in any base. **Trigonometric functions:** • Trigonometric relationships and functions can be used to model real-world phenomenon. • Indirect measurements of lengths and angles can be used to solve a variety of problems. • Domain must be limited to finding the inverse of a trigonometric function. • Inverse functions must be used to find solutions in some modeling problems. • A circle is a set of points that can be defined by an equation. This measurement is expressed in radians rather than degrees. • Students extend the domain of trigonometric functions using the unit circle. • Students establish a way to measure angles with respect to arc length. • The trigonometric functions are extended to all real numbers to describe rotations around the unit circle. • Sine, cosine, and tangent functions can be defined using the unit circle. • Our world is periodic. The amount of sunlight a city receives on a given day, high and low tides are all real life instances where sinusoids explain and model real life phenomena.	17. What are periodic functions and why is modeling them so important? 18. Why is the Theorem of Pythagoras so essential in trigonometry? 19. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. 20. How can the graphs of the sine, cosine, tangent functions and their inverses be compared? 21. How can you use the addition and subtraction formulas for sine, cosine, and tangent to solve problems? 22. What do we do to find the inverses of trigonometric functions? 23. How can you solve trigonometric equations using the inverse functions? 24. What are the period, amplitude, and midline of the graph of a trigonometric function? 25. How can technology be used to evaluate solutions of trigonometric functions?	• Radian Measure • Rational functions • Reflection over the x and y-axis • Relative Minimum • Restricted domain • Sine, cosine, tangent • Sinusoidal graphs • Secant, cosecant, tangent • Step function • Symmetries • Transformations • Trigonometric functions • Vertical/ horizontal shifts

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Illustrative Mathematics • Functions: https://www.illustrativemathematics.org/standards/hs • Graphic Representations of the Real Life Situations http://graphingstories.com/ • Applications of Trigonometry: www.math12.com • Prentice Hall Algebra Two Online resources: http://www.phschool.com/atschool/phmath07/program_page_hs.html **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2nd edition, Foerster, Key Curriculum (2007)	**Unit 2 is largest unit** which includes the variety of major topics such as interpreting and building functions including the logarithmic and trigonometric Functions. Students will be analyzing functions, graphing with transformations, comparing/contrasting functions graphs and solve real-world problems. Students are required to understand families of functions and the inverse of those functions. Students must be familiar with the concept and formal definition of inverse functions, namely that if $f \circ g(x) = g \circ f(x) = x$, then $f(x)$ and $g(x)$ are inverses of one another. Teachers should first work with evaluating functions, then composing general functions and finally composing inverse functions. Once students have mastered the composition of inverse functions, they should be made to derive the inverse functions and prove that they have found the inverse by using the above definition. Students should recall parent functions $f(x)$ and then explore the effect of $f(x)+k$, $f(x+k)$, $kf(x)$, $f(kx)$ on the graph for all k. The mathematical progressions demand that students are fluent with the parent functions and can use them quickly to determine the graph of transformed functions. Students will explore the relationship between functions and their inverses on the same coordinate plane. They will use that understanding to then explain the connection between the line of symmetry of the two functions and the algebraic method of letting $f(x) = x$ and $x = f^{-1}(x)$ to solve for the inverse function $f^{-1}(x)$. Students should then come to understand why a function needs to be one-to-one in order to have an inverse and then why it is necessary and possible to restrict a domain on a function to create an invertible function. Provide visual examples of transformed functions while manipulating different constants in the function parameters. Have students use technology to manipulate the parameters of the functions and record how the parameters affect the graphs and tables of the functions.	SBAC – http://www.smarterbalanced.org/ PARCC - http://www.parcconline.org/sites/parcc/files/HighSchoolAlg2Math3-GraphsofFunctions.pdf http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

	Use a compass and straightedge to explore a unit circle with a fixed radius of 1. Help students to recognize that the circumference of the circle is 2π, which represents the number of radians for one complete revolution around the circle. Students can determine that, for example, $\pi/4$ radians would represent a revolution of 1/8 of the circle or 45°. Students can examine how a counterclockwise rotation determines a coordinate of a particular point in the unit circle from which sine, cosine, and tangent can be determined. Have students explore real-world examples of periodic functions; such as: average high (or low) temperatures throughout the year, the height of ocean tides as they advance and recede, and the fractional part of the moon that one can see on each day of the month. Graphing some real-world examples can allow students to express the amplitude, frequency, and midline of each. • *Teachers might find the following strategies helpful:* **Cooperative learning**: Engage all students by using a variety of differentiation strategies including but not limited to questioning techniques, wait time, Think-Pair-Share, peer tutoring, small groups collaboration Students are required to understand families of functions and the inverse of those functions. **Checking for understanding and reflecting on students' background knowledge:** Use a variety of strategies to frequently check for understanding such as small white boards, hand signals (thumbs up/thumbs down), parking lot questions, etc. Teachers make connections to students' prior knowledge. **Problem-solving and abstract reasoning:** Analyze the data, compare/contrast, use counterexamples, construct plausible arguments, make conclusions, justify different ways to solve a problem and communicate to others. **Quick write:** Explain the process and the solution by using academic language and key vocabulary. **Modeling and solving real-world problems:** Apply algebraic skills and knowledge to solve a variety of engaging/ relevant problems to make assumptions, analyze the data, derive to solutions and draw viable conclusions. **Technology-enhanced instruction:** Utilize graphing calculators, spreadsheets, computer algebra systems, statistical packages and other appropriate software. **Project-based learning:** Use a variety of problem-solving	

	assignments such as creating/ solving word problems and the culminating unit tasks.	

LANGUAGE GOALS

Writing:

1) Students will explain and justify in writing the behavior of the function as it approaches horizontal and vertical asymptotes.
Example: As the function approaches positive infinity along the x-axis, the graph of the function approaches the horizontal asymptote from above.

2) Students will explain (in writing and orally) the effects of transformations on a function and test that understanding for all parent functions.
Example: The transformation $f(x+a)+b$, moves the parent function $-a$ units in the horizontal direction and b units in the vertical direction.

3) Students will compare and contrast (in writing and orally) the differences and similarities between linear, polynomial, and exponential functions.
Example: All three functions increase as x increases. Polynomial and exponential functions are curves and the linear function is a line. Exponential functions will increase at a faster rate than polynomial functions.

4) Students will write about the relationship between the inverse of functions and the concept of rotating the axes about the line of symmetry to determine the inverse function.
Example: The inverse function can be determined by rotating the function of the graph about the line of symmetry. This is algebraically equivalent to interchanging the x and y values in a function and solving for y.

5) Students will write about how functions can be used to in real life to facilitate repeated algorithms.
Example: Computers often make use of functions to run programs i.e. clicking on the icon for Internet Explorer will run a function to launch a program that connects the modem to the internet and opens a screen to a preselected page.

6) Students will explain in writing how to prove and apply the Laws of Sines and Cosines using technical vocabulary in complex sentences.

7) Students will explain (in writing and orally) the terms and definitions of the trigonometric functions; conic sections; and complex numbers.
Example: To find the (<u>amplitude</u>) of the function, I can first find the (<u>midline</u>) and then find the distance to the (<u>maximum or minimum</u>) of the graph.

Listening and Speaking:

1) Students will participate in class discussions using specific vocabulary related to transformations and functions.

2) Students will explain and justify (orally) how to graph a function to a partner as well as restating and summarizing their partner's explanation.
Example: First I _____ because _____, second I _____ because_____,…

Reading:

1) Students will identify the relevant information and details in a passage and create a single function that represents a composition out of many subparts.

PERFORMANCE TASKS

<u>Precalculus Enhanced with Graphing Utilities</u>, Sullivan & Sullivan, 4th Edition (2005), **ISBN-10:** 0131490923

F-IF.7d

- Population Model, Page 197, # 53 and 54
- Cost of a Can, Page 210, # 61
- **Waves, Chapter Project**, Page 515, # 1
- Discussion and Writing, Page 270, # 84-90

<u>Precalculus Graphical, Numerical, Algebraic</u>, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education 2007

F-IF.7d

- Designing a Cardboard Box, Page 265, # 59
- Industrial Design, Page 272, # 94 and 95
- Designing a Juice Can, Page 265, # 61

Illustrative Mathematics:

1) F-IF.9 Throwing Baseballs : https://www.illustrativemathematics.org/illustrations/1279
2) F-BF.1 Compounding with a 5% Interest Rate: https://www.illustrativemathematics.org/illustrations/572
3) F-BF.3 Transforming the graph of a function : http://www.illustrativemathematics.org/illustrations/742
4) F-BF.3 Building an Explicit Quadratic Function by Composition: www.illustrativemathematics.org/illustrations/744
5) F-LF.4 Carbon 14 Dating : https://www.illustrativemathematics.org/illustrations/369
6) F-TF. 1 Bicycle Wheel: https://www.illustrativemathematics.org/illustrations/1873
7) F-TF.5 As the Wheel Turns: https://www.illustrativemathematics.org/illustrations/595
8) F-TF. 5 Foxes and Rabbits 2 : https://www.illustrativemathematics.org/illustrations/816
9) F-TF.5 Foxes and Rabbits 3: https://www.illustrativemathematics.org/illustrations/817
10) F-TF.5 Exploring Sinusoidal Functions F-TF.5 https://www.illustrativemathematics.org/illustrations/1647

DIFFERENTIATION

FRONT LOADING	ACCELERATION	INTERVENTION
• Have students recall how to graph by hand linear, quadratic and cubic functions from a table of values and then understand how to graph all parent functions. • Get the students to explain how to solve quadratic equations by the quadratic formula and completing the square. • Engage students in an activity that would involve comparing linear functions with quadratics functions, and then quadratics functions and exponential functions. • Involve students in the processes required to solve equations and start to discuss the concept of inverse functions. • Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations.	• Students work in small groups with a curriculum that is conceptually demanding as well as rigorous due to the speed at which the course moves and the concepts covered. Students collaborate and concentrate on tasks for extended periods of time, to contribute to discussions, to predict and test their predictions. • The assessments for advanced students will demand the ability to apply learned concepts to solving abstract or real world problems or summarize the patterns/ concepts learned. Students will use the "Socratic Method" for posing questions to discover connections, patterns and structure. • Students learn about the modeling of real world data with polynomial functions, rational functions, exponential functions, radical functions, logarithmic functions, and sinusoidal functions. They explore in depth the various characteristics of functions, i.e.	• Reflect on students prior knowledge of the following Algebra 1 topics: • Radicals and exponents, rational expressions and equations, operations with polynomials and the basic graphic techniques. • Review the difference between independent events and dependent variables. • Review the difference between real and complex roots and the operations with complex numbers. • Review how to create tables of values and to use those values to generate the graph of the function. • Review key vocabulary words from unit 1. • Allow students to use technology to

• Involve students in the discussion on zeros of polynomial functions and their roots/zeros. • Have students recall the properties of exponents including rational exponents. • Check students understanding of the geometric transformations such as translations and reflections. • Have students recall the trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	rates of change, concavity, inverses, continuity, discontinuity and asymptotes. Students further explore functions in terms of composite and inverse functions, their transformations and periodicity. • Students work on projects to apply these concepts to real-world problems by creating equations and exploring the graphs of those equations using technology application to determine which parts of the graph are relevant to the problem context.	quickly generate a table of values after they have shown some skill in evaluating expressions by hand. • Using technology, students work in small groups to graph different functions and compare/contrast the graphs and make conclusions.

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